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Nonuniversality in microbranching instabilities in rapid fracture

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The apparent similarity of microbranching instabilities in different brittle materials gave rise to a widely held belief that many aspects of the postinstability physics were universal. We propose that the physics determining the typical length and time scales characterizing the postinstability patterns differ greatly from material to material. We offer a scaling theory connecting the pattern characteristics to material properties (like molecular weight) in brittle plastics like *PMMA*, and stress the fundamental differences with patterns in glass which are crucially influenced by three-dimensional dynamics. In both cases the present *ab initio* theoretical models are still too far from reality, disregarding some fundamental physics of the phenomena.

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More than a decade of high precision experiments on rapid crack propagation in brittle materials has ignited the interest of the physics community [1]. It appears that one important prediction of "linear elasticity fracture mechanics" [2], stating that cracks will accelerate smoothly as they lengthen until they reach their asymptotic velocity (bounded either by the loading conditions or by the Rayleigh speed c_R), is in fundamental contradiction with experiments. In thin brittle plates of width W (see Fig. 1; W is in the z direction, with the length in x and the height in y much larger than W), at some finite fraction v_c of the Rayleigh speed, the smooth dynamics of the crack evolving in the x direction is marred by the appearance of microbranches in the x-y plane that do not cut through the whole width W. This dynamical instability was observed in many different materials, including glass, plastics, and gels. The experimental analysis [3,4] of the postinstability features (geometry of side branching, velocity fluctuations, etc.), as well as theoretical models of the phenomenon [5], stressed the apparent universality, as if the dynamics are invariant to the material. The aim of this Rapid Communication is to point out that the universality may have been overstressed.

The microbranches have a typical length ℓ_b, a typical width $\Delta z \leqslant W$, and they appear to recur with an average periodicity. We will refer to this recurrence scale as "the scale of noisy periodicity" Δx. In glass, this scale appears to range between the measurement resolution (~ 1 μm) and a few mm [7], while in *PMMA* between tens and hundreds micrometers [6], depending on the material properties and the experimental conditions. We argue in this Rapid Communication that the nature of this length scale and the physics that determine it are nonuniversal.

As is well known, linear elasticity fracture mechanics has no typical scale, and thus the appearance of such a scale is yet another demonstration of the need to modify this theory in the context of dynamic fracture. Indeed, there exist concepts in the classical theory of crack propagation that can be used to form a length scale. Recall that in the classical theory one asserts that as a crack of length L propagates at velocity v under loading conditions σ, the energy $G(\sigma,L,v)$ released from the stressed material streams into the tip region where it is compensated by the total dissipation $\Gamma(v)$,

$$G(\sigma,L,v) = \Gamma(v). \tag{1}$$

The dissipation function $\Gamma(v)$ is not computable from elasticity theory, but it can be measured in experiments [essentially by using Eq. (1)]. Equipped with this function and the Young modulus E of the material we can form a (velocity dependent) scale from

$$\ell(v) \equiv \Gamma(v)/E. \tag{2}$$

To get a rough estimate of the resulting scale, we use the data for *PMMA* determined in [1] near the onset of the microbranching instability: $\Gamma(v_c) \simeq 3000$ J/m^2 and $E \simeq 3 \times 10^9$ N/m^2. This predicts $\ell(v)$ in the μm range; a similar estimation for glass yields a nm scale. We thus understand that the scale computed in Eq. (2) may be relevant for *PMMA*, but appears utterly out of range for glass, where the observed scales of noisy periodicity must be sought elsewhere, as we discuss below. Given information about the largest stress value that marks the breakdown of the linear theory, say σ_m, another length scale can be constructed as

$$\tilde{\ell}(v) \equiv E\Gamma(v)/\sigma_m^2. \tag{3}$$

We estimate σ_m by the yield stress of the material. For the materials under discussion the resulting length scale is typically two order of magnitude larger than (2), putting it right in the range of scales of noisy periodicity for *PMMA*, but still off range for glass. We continue now to assess the relevance of (2) or (3) to *PMMA*. Obviously, the dissipation $\Gamma(v)$ in *PMMA* should be a strong function of the molecular

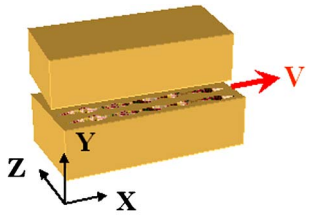

FIG. 1. (Color online) Typical geometry of a dynamically generated crack, with the side branches that result from the instability leaving their mark on the faces of the crack.

weight M of the polymers that make the material, due to the increased density of entanglements of the polymer chains. Indeed, a calculation of the M dependence of Γ in *quasistatic* conditions exists [8], with the resulting prediction

$$\Gamma(v=0) \sim \exp(-M_0/M), \tag{4}$$

where M_0 is a constant. On the other hand, the Young modulus E and the yield stress were shown [9,10] to be M independent.

The length (2) [or (3)] will be dressed by dynamical effects, which we introduce in the form $\ell(v)=e^{-M_0/M}g(v/c_R)$. To proceed, we adopt the scaling assumption that $\ell(v)$ is the only typical scale in the problem, characterizing the length ℓ_b of microbranches in the x-y plane as well as Δx in the x-z plane. In addition, we will assert that in *PMMA* (in contrast to glass, see below for further discussion) the relevant length scales are determined by a competition between the side microbranch and the main crack, without important dynamical coupling to the third dimension. In this respect the mechanism underlying the repetitive nature of the microbranching process in *PMMA* is qualitatively similar to the two-dimensional branching model developed recently in [11]. In this model, the local competition between the side branch and the main crack temporarily slows the main crack below the relevant branching velocity. When the main crack regains the critical velocity another side branching event occurs (see Fig. 7 in [11]). It was found (see Fig. 6 in [11]) that the velocity dependence of ℓ_b has the following form:

$$\ell_b(v) = \ell_b(v_c)[1 + \alpha(v - v_c)/c_R]. \tag{5}$$

Here $\ell_b(v_c)$ is the finite length of the side branch which occurs at the threshold and α is a dimensionless coefficient. In [11] this length was introduced by hand to represent the scale over which linear elasticity theory is not applicable (the so-called process zone); here we take it as the undressed length proportional to $\exp(-M_0/M)$. Collecting our assumptions together we present the prediction for the scale of noisy periodicity and the length of the microbranches in *PMMA*,

$$\Delta x(v,M) \sim \ell_b(v,M) \sim e^{-M_0/M}[1 + \alpha(v - v_c)/c_R]. \tag{6}$$

Note that if indeed the scale $\ell_b(v_c)$ is physically identified with the process zone, it may have a velocity dependence of its own. If this velocity dependence is linear, the prediction (6) remains unchanged.

It is possible to test this major prediction against available experiments. The influence of the mean molecular weight M on the scale of noisy periodicity in *PMMA* was studied in [12]. Figure 2 displays their experimental data for $M\ln(\Delta x)$ as a function of M (taken from Table 1 in [12]). The almost perfect linearity of the graph lends a strong support to the M dependence in our scaling law. Next, the branch length at a fixed mean molecular weight as a function of the normalized mean velocity for *PMMA* is shown in Fig. 3. It is clear that ℓ_b is linear in v as predicted by our scaling relation. Moreover, the fit indicates the existence of a finite length $\ell(v_c) \approx 30\ \mu$m as predicted by the scaling law. This finite length is determined uniquely in this graph once v_c is known. The value of $\ell(v_c)$ is confirmed in [6]. Moreover, dividing $\ell(v)$ by v one gets a typical time scale (say for $v=500$ m/s we get a typical time of 0.7 μs, in agreement with [6].)

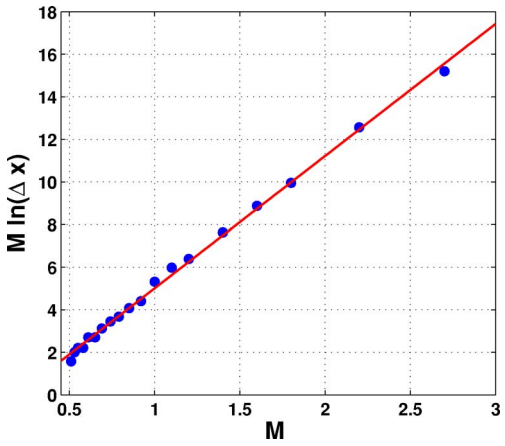

FIG. 2. (Color online) $M\ln(\Delta x)$ as a function of M (measured in units of 10^5). The filled circles are the experimental data taken from [12] (see Table 1 in [12]), the solid line is a linear fit with $M_0 \approx 1.2$, and the proportionality constant in Eq. (6) is $\simeq 500\ \mu$m.

In summary, the time and length scales of the noisy periodicity in *PMMA* appear to be adequately described by a scaling theory in which the main scale depends on the material properties Γ, E, and possibly σ_m, dressed by dynamical effects which are not fundamentally dependent on a coupling to the dynamics in the third dimension. There is nothing like that in glass. First, we remarked before that the length scales (2) and (3) are in 1–100 nm range, whereas the noisy periodicity in glass appears on scales up to a few mm. Moreover, we will point out now that there is no typical scale of noisy periodicity in glass experiments; the distances between microbranching events can vary over three orders of magnitude (from the measurement resolution to the scale of the width W), and they are determined by a dynamical mechanism that rests crucially on the dynamics of the crack front in the narrow third dimension along the width W.

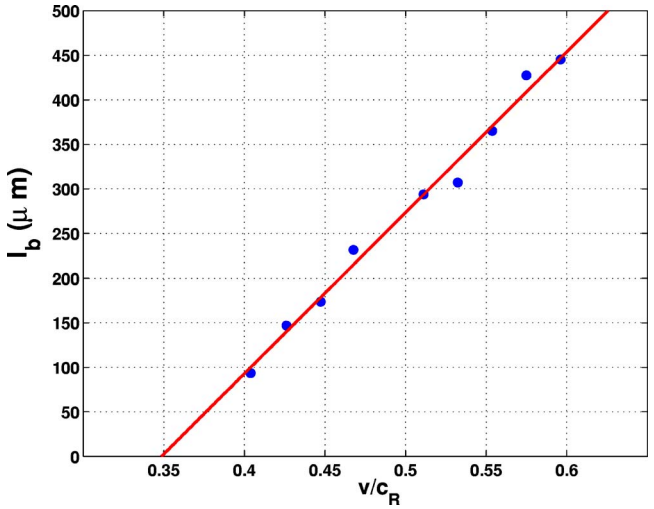

FIG. 3. (Color online) The branch length ℓ_b as a function of the mean normalized velocity for *PMMA*. Data taken from [3]. The solid line represent the liner fit with $\alpha \approx 60$, $v_c \approx 0.365 c_R$, and $\ell(v_c) \approx 30\ \mu$m.

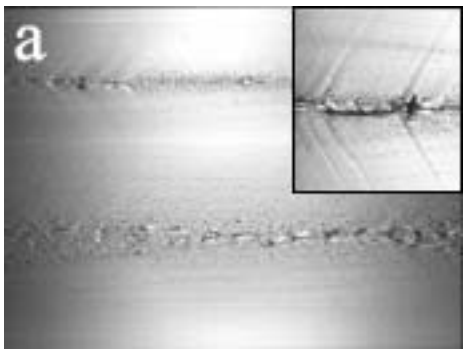

FIG. 4. The correlated microbranches in glass samples. Each series of microbranches is localized in a strip of width Δz, and has a noisy periodicity $\Delta x \simeq 4$–$5\Delta z$, adapted from [7].

In glass, once a microbranch forms at a location z along the width of the sample and with a width Δz, the next branching events fall very precisely along a strip of width Δz localized at the same location z; see Fig. 4 as an example. The crucial phenomenon that is responsible for these highly correlated structures is the interaction of the crack front with spatial heterogeneities (regions of variable Γ). These can generate nondecaying waves that travel with velocity c_{FW} (relative to the heterogeneity) on the front as it propagates [13]. Plane numerical simulations [14] have shown that the interaction of the front with an asperity leads to a velocity overshoot *exactly ahead* of the interaction site, see Fig. 5.

Since the creation of a microbranch increases Γ locally, one expects the generation of front waves, as indeed was observed in glass and can be seen in the inset of Fig. 4. The resulting velocity overshoot just ahead of the microbranching event was interpreted by Sharon, Cohen, and Fineberg [7] as the source of the well-defined lines of microbranches in glass. In this picture, the creation of a microbranch reduces

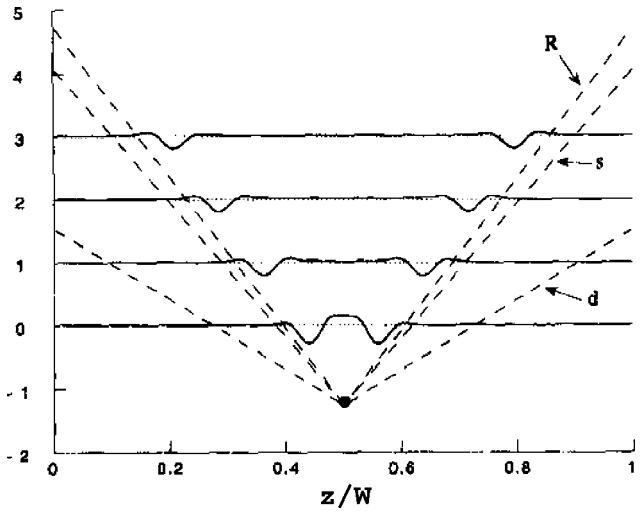

FIG. 5. The front waves generated by the interaction of the front with an asperity. The front velocity profile is plotted at constant time intervals. Note the velocity overshooting just ahead of the asperity. d,s,R are the cones generated by the dilatational, shear, and Rayleigh waves, respectively. The relation $c_{FW} \approx c_R$ is apparent, adapted from [14].

locally the velocity of the main crack below the microbranching velocity, but this velocity is reached again due to the velocity overshoot just ahead. The typical time τ for this process is the time needed for the front wave to travel a distance of the order of the width of the microbranch Δz. Since the velocity of the front wave along the front is $\sqrt{c_{FW}^2 - v^2}$ one obtains

$$\tau \sim \Delta z(v)/\sqrt{c_{FW}^2 - v^2}. \tag{7}$$

Therefore, the noisy periodicity scale Δx is estimated as

$$\Delta x(v) \sim \Delta z(v) v/\sqrt{c_{FW}^2 - v^2}. \tag{8}$$

Since the combination $v/\sqrt{c_{FW}^2 - v^2}$ is a slowly increasing function of v $[0.94 c_R \leqslant c_{FW}(v) \leqslant c_R]$ in the relevant range of velocities, one expects the ratio $\Delta x(v)/\Delta z(v)$ to be nearly constant and this is indeed the case as shown in [7], where the relation is stated to be $\Delta x \simeq 4$–$5\Delta z$. The point that we want to stress here is that in contradistinction with *PMMA*, in glass *there exist no typical scale for the noisy periodicity*. $\Delta x(v)$ can be *anything*, depending on the width of the first microbranch $\Delta z(v)$. After that the dynamics will sustain repeated microbranches every $\Delta x \simeq 4$–$5\Delta z$. The only thing that can happen is that $\Delta z(v)$ might increase as a function of v, which is the last issue that we discuss.

As noted earlier, Eq. (1) predicts that cracks accelerate smoothly towards their asymptotic velocity. A crucial assumption leading to this prediction is that a single crack whose front is essentially a point, can expend the extra energy flowing to its tip, by adjusting its velocity and increasing the kinetic energy of its surroundings. This assumption holds up to v_c, but breaks down for $v > v_c$. At the onset of instability, a *multiple* crack state is formed by repetitive frustrated microbranching events, and these microbranches break the translational invariance in the third dimension, occupying a finite width $\Delta z \leqslant W$. The essence of the instability is that the additional energy supply is expended on creating more surface area per unit crack extension, even at a constant velocity. This process introduces new *dynamic variables* to the problem: the number of coexisting branches n, the morphology of the branches in the x-y planes and their width Δz. All these variables contribute to the additional energy dissipation per unit crack extension. It was shown [15] that the total surface energy created per unit crack extension is proportional to the total energy release rate, implying that in this regime the fracture energy $\Gamma(v)$ is constant, independent of the velocity. Assuming that Δz characterizes the width of the microbranches as long as they exist, we can write

$$WG(v) = \Delta z(v) f(n(v), \ldots)\Gamma + W\Gamma, \tag{9}$$

where the right-hand side is a sum of the dissipative contributions from the microbranches as well as from the main crack, respectively. The function $f(n(v), \ldots)$ depends on the number of coexisting microbranches, their morphology, and maybe other dynamic variables. The last relation can be rewritten as

$$\Delta z(v) = W[G(v)/\Gamma - 1]/f(n(v), \ldots). \tag{10}$$

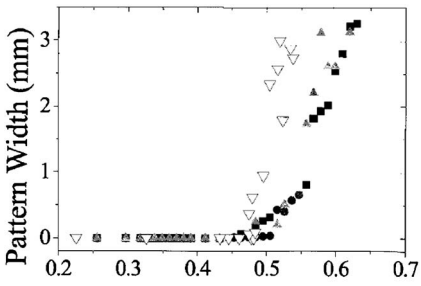

FIG. 6. The pattern width Δz as a function of the normalized mean velocity v/c_R for the *PMMA* (filled symbols) and glass (empty symbols), adapted from [3].

Although $f(n(v),\ldots)$ is not known in adequate detail, we can still draw a relevant qualitative conclusion. Since $G(v)$ is known from experiments to be a strongly increasing function of v [15], we can safely expect that $\Delta z(v)$ is also a strongly increasing function of v. This expectation is supported by the data; the dependence of Δz on the mean velocity v for both *PMMA* and glass is shown in Fig. 6. We stress that in glass the increase in $\Delta z(v)$ *must* affect the scale of noisy periodicity, whereas in the *PMMA* it does not. In glass, the dynamics along the third dimension are *strongly* coupled to the dynamics in the longitudinal direction (by the front waves mechanism), and the dynamic length scale Δz controls all the other length scales. In *PMMA*, the dynamics along the third dimension are only *weakly* coupled to the dynamics in the longitudinal direction, where the length scales are determined by material properties and the elastodynamic competition between the microbranches and the main crack. We note that in *PMMA* the front waves are decaying due to the condition $\partial\Gamma/\partial v > 0$ Ref. [13].

In summary, models of dynamic fracture can hope to adequately describe the dynamic instabilities only if the physics that determines the fundamental scales were incorporated. In glass, without adequate treatment of the third dimension and the physics of the crack front, not much insight into the microbranching process can be gained. In *PMMA* the coupling to the third-dimension might be weaker, but one should resolve how the material parameters appear in the continuum description. Completely different scaling laws appear in these two materials due to the different mechanisms that control the postinstability physics; probably other materials may increase the richness of the phenomenology. It might be worthwhile to stress less the presence of universal features and more the interesting physics that can be learned even with simple scaling theories.

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[1] See M. Marder and J. Fineberg, Phys. Rep. **313**, 1 (1999); and references therein.
[2] L. B. Freund, *Dynamic Fracture Mechanics* (Cambridge, London, 1998).
[3] E. Sharon and J. Fineberg, Philos. Mag. B **78**, 243 (1998).
[4] A. Livne, G. Cohen, and J. Fineberg, Phys. Rev. Lett. **94**, 224301 (2005).
[5] I. S. Aranson, V. A. Kalatsky, and V. M. Vinokur, Phys. Rev. Lett. **85**, 118 (2000); H. Henry and H. Levine, Phys. Rev. Lett. **93**, 105504 (2004).
[6] E. Sharon and J. Fineberg, Phys. Rev. B **54**, 7128 (1996).
[7] E. Sharon, G. Cohen, and J. Fineberg, Phys. Rev. Lett. **88**, 085503 (2002).
[8] A. G. Mikos and N. A. Peppas, J. Chem. Phys. **88**, 1337 (1988).
[9] R. P. Kusy and D. T. Turner, Polymer **17**, 161 (1976).
[10] P. I. Vincent, Polymer **1**, 425 (1960).
[11] E. Bouchbinder, J. Mathiesen, and I. Procaccia, Phys. Rev. E **71**, 056118 (2005).
[12] R. P. Kusy and D. T. Turner, Polymer **18**, 391 (1977).
[13] S. Ramanathan and D. S. Fisher, Phys. Rev. Lett. **79**, 877 (1997).
[14] J. W. Morrissey and J. R. Rice, J. Mech. Phys. Solids **46**, 467 (1998).
[15] E. Sharon, S. P. Gross, and J. Fineberg, Phys. Rev. Lett. **76**, 2117 (1996).